OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2640

Mechanics 4

Thursday **22 MAY 2003** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\,\text{m}\,\text{s}^{-2}$.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

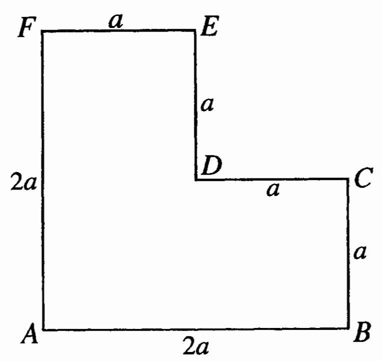
This question paper consists of 4 printed pages.

1 A propeller shaft has constant angular acceleration. It turns through 160 radians as its angular speed increases from 15 rad s^{-1} to 25 rad s^{-1}. Find

(i) the angular acceleration of the propeller shaft, [2]

(ii) the time taken for this increase in angular speed. [2]

2



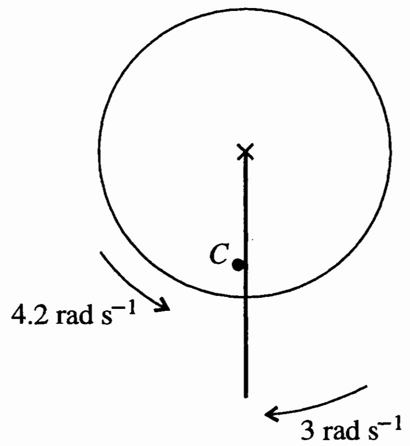
The diagram shows a uniform lamina *ABCDEF* in which all the corners are right angles. The mass of the lamina is $3m$.

(i) Show that the moment of inertia of the lamina about *AB* is $3ma^2$. [3]

(ii) Find the moment of inertia of the lamina about an axis perpendicular to the lamina and passing through *A*. [2]

3 A uniform rod, of mass 0.75 kg and length 1.6 m, rotates in a vertical plane about a fixed horizontal axis through one end. A frictional couple of constant moment opposes the motion. The rod is released from rest in a horizontal position and, when the rod is first vertical, its angular speed is 3 rad s^{-1}.

(i) Find the magnitude of the frictional couple. [4]



A disc is rotating about the same axis. The moment of inertia of the disc about the axis is 0.56 kg m^2. When the rod is vertical, the disc has angular speed 4.2 rad s^{-1} in the opposite direction to that of the rod (see diagram). At this instant the rod hits a magnetic catch *C* on the disc and becomes attached to the disc.

(ii) Find the angular speed of the rod and disc immediately after they have become attached. [3]

4 A cruise ship C is sailing due north at a constant speed of 12 m s^{-1}. A boat B, initially 2000 m due west of C, sails with constant speed 11 m s^{-1} on a straight line course which takes it as close as possible to C.

(i) Find the bearing of the direction in which B sails. [4]

(ii) Find the shortest distance between B and C in the subsequent motion. [4]

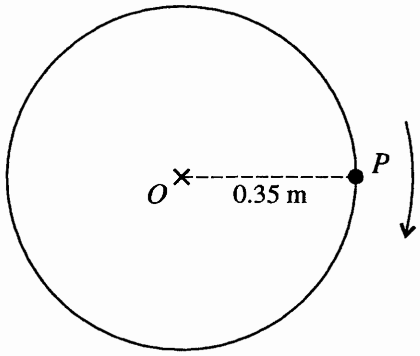
5 The region bounded by the x-axis, the line $x = 8$ and the curve $y = x^{\frac{1}{3}}$ for $0 \leqslant x \leqslant 8$, is rotated through 2π radians about the x-axis to form a uniform solid of revolution. The unit of length is the metre, and the density of the solid is 350 kg m^{-3}.

(i) Show that the mass of the solid is 6720π kg. [3]

(ii) Find the x-coordinate of the centre of mass of the solid. [3]

(iii) Find the moment of inertia of the solid about the x-axis. [4]

6



A wheel consists of a uniform circular disc, with centre O, mass 0.08 kg and radius 0.35 m, with a particle P of mass 0.24 kg attached to a point on the circumference. The wheel is rotating without resistance in a vertical plane about a fixed horizontal axis through O (see diagram).

(i) Find the moment of inertia of the wheel about the axis. [3]

(ii) Find the distance of the centre of mass of the wheel from the axis. [2]

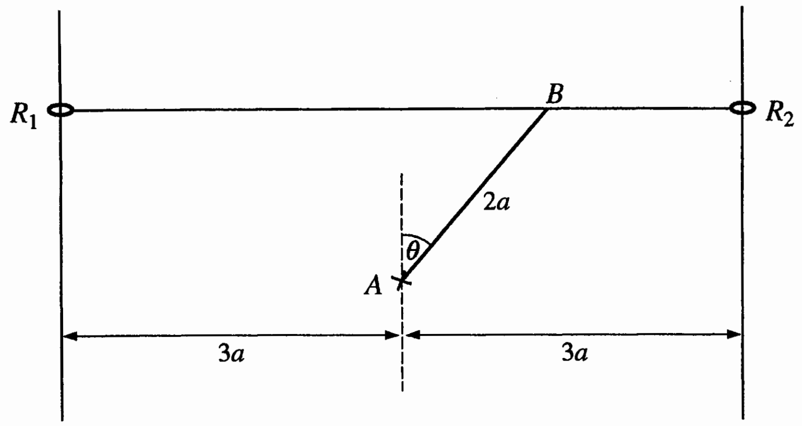
At an instant when OP is horizontal and the angular speed of the wheel is 5 rad s^{-1}, find

(iii) the angular acceleration of the wheel, [2]

(iv) the magnitude of the force acting on the wheel at O. [6]

[Question 7 is printed overleaf.]

7

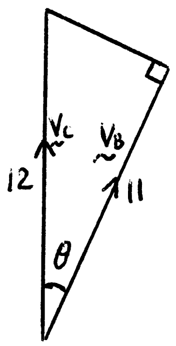
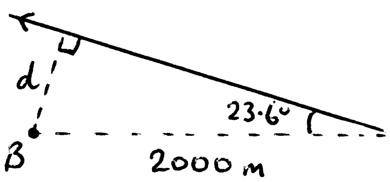


A uniform rod AB, of mass m and length $2a$, is pivoted to a fixed point at A and is free to rotate in a vertical plane. Two fixed vertical wires in this plane are a distance $6a$ apart and the point A is half-way between the two wires. Light smooth rings R_1 and R_2 slide on the wires and are connected to B by light elastic strings, each of natural length a and modulus of elasticity $\frac{1}{4}mg$. The strings BR_1 and BR_2 are always horizontal and the angle between AB and the upward vertical is θ, where $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$ (see diagram).

(i) Taking A as the reference level for gravitational potential energy, show that the total potential energy of the system is

$$mga(1 + \cos\theta + \sin^2\theta).$$ [5]

(ii) Given that $\theta = 0$ is a position of stable equilibrium, find the approximate period of small oscillations about this position. [8]

1 (i)	$25^2 = 15^2 + 2\alpha \times 160$ $\alpha = 1.25 \text{ rad s}^{-2}$	M1 A1 **2**	Use of appropriate formula
(ii)	$160 = \frac{1}{2}(15+25)t$ $t = 8 \text{ s}$	M1 A1 **2**	Or $25 = 15 + 1.25t$ etc
2 (i)	$I_{AB} = \frac{4}{3}(2m)a^2 + \frac{4}{3}(m)(\frac{1}{2}a)^2$ $= \frac{8}{3}ma^2 + \frac{1}{3}ma^2$ $= 3ma^2$	B1B1 B1 (ag) **3**	
	OR $\frac{4}{3}(2m)(\frac{1}{2}a)^2 + \left\{\frac{1}{3}m(\frac{1}{2}a)^2 + m(\frac{3}{2}a)^2\right\}$ B1B1 $= \frac{2}{3}ma^2 + \frac{7}{3}ma^2 = 3ma^2$ B1		
	OR $\frac{4}{3}(4m)a^2 - \left\{\frac{1}{3}m(\frac{1}{2}a)^2 + m(\frac{3}{2}a)^2\right\}$ B1B1 $= \frac{16}{3}ma^2 - \frac{7}{3}ma^2 = 3ma^2$ B1		
(ii)	$I = I_{AB} + I_{AF}$ $= 6ma^2$	M1 A1 **2**	Use of perpendicular axes rule
3 (i)	$I = \frac{4}{3} \times 0.75 \times 0.8^2 \quad (= 0.64)$ $-C(\frac{1}{2}\pi) = \frac{1}{2}I \times 3^2 - 0.75 \times 9.8 \times 0.8$ $C = 1.91 \text{ N m}$	B1 M1 A1 A1 **4**	Equation involving WD, KE, PE Accept $\frac{6}{\pi}$
(ii)	$(0.64 + 0.56)\omega = 0.56 \times 4.2 - 0.64 \times 3$ $\omega = 0.36 \text{ rad s}^{-1}$	M1 A1 ft A1 **3**	Conservation of angular momentum
4 (i)	 $\cos\theta = \frac{11}{12}$ $\theta = 023.6°$	M1 A1 M1 A1 **4**	Relative velocity perpendicular to $\mathbf{v}_B$ Correct velocity triangle
(ii)	As viewed from *B*  $d = 2000 \sin 23.6$ $= 799 \text{ m}$	M1 A1 ft M1 A1 **4**	Use of relative velocity Correct diagram with shortest distance indicated Accept 799 to 801

5 (i)	$V=\int_0^8 \pi(x^{\frac{1}{3}})^2\,dx$ $=\pi\left[\frac{3}{5}x^{\frac{5}{3}}\right]_0^8$ $=\frac{96}{5}\pi$ Mass is $350\times\frac{96}{5}\pi=6720\pi$ kg	M1 A1 A1 (ag) **3**	 For $\frac{3}{5}x^{\frac{5}{3}}$
(ii)	$\int xy^2 dx=\int_0^8 x(x^{\frac{1}{3}})^2 dx$ $=\left[\frac{3}{8}x^{\frac{8}{3}}\right]_0^8=96$ $\bar{x}=\frac{\int xy^2 dx}{\int y^2 dx}=\frac{96}{\frac{96}{5}}$ $=5$	M1 M1 A1 **3**	
(iii)	$I=\int\frac{1}{2}(\rho\pi y^2)y^2 dx$ $=\frac{1}{2}\times350\pi\int_0^8(x^{\frac{1}{3}})^4 dx$ $=175\pi\left[\frac{3}{7}x^{\frac{7}{3}}\right]_0^8$ $=9600\pi\quad(=30\,200)$ kg m^2	M1 A1 A1 A1 **4**	Integration of y^4 For $\frac{3}{7}x^{\frac{7}{3}}$
6 (i)	$I=\frac{1}{2}\times0.08\times0.35^2+0.24\times0.35^2$ $=0.0343$ kg m^2	B1B1 B1 **3**	
(ii)	$(0.08+0.24)\bar{x}=0+0.24\times0.35$ $\bar{x}=0.2625$ m	M1 A1 **2**	
(iii)	$0.24\times9.8\times0.35=I\alpha$ $\alpha=24$ rad s^{-2}	M1 A1 **2**	Or $0.32\times9.8\times0.2625=I\alpha$ Accept $2.45g$ *(but omission of g is M0)*
(iv)	$H=mr\omega^2$ $=0.32\times0.2625\times5^2\quad(=2.1)$ $mg-V=mr\alpha$ $V=0.32\times9.8-0.32\times0.2625\times24\quad(=1.12)$ Magnitude $\sqrt{H^2+V^2}=2.38$ N	M1 A1 ft M1 A1 ft M1A1 **6**	

7 (i)	GPE is $mga\cos\theta$	B1	
	EPE in BR_1 is $\frac{1}{2}\frac{\frac{1}{4}mg}{a}(3a+2a\sin\theta-a)^2$ $=\frac{1}{2}mga(1+\sin\theta)^2$	M1A1	
	EPE in BR_2 is $\frac{1}{2}\frac{\frac{1}{4}mg}{a}(3a-2a\sin\theta-a)^2$ $=\frac{1}{2}mga(1-\sin\theta)^2$	A1	
	EPE $=\frac{1}{2}mga(1+2\sin\theta+\sin^2\theta+1-2\sin\theta+\sin^2\theta)$ $=mga(1+\sin^2\theta)$		
	Total PE, $V=mga(1+\cos\theta+\sin^2\theta)$	A1 (ag) **5**	
(ii)	KE is $\frac{1}{2}(\frac{4}{3}ma^2)\dot{\theta}^2$	M1A1	
	$\frac{2}{3}ma^2\dot{\theta}^2+mga(1+\cos\theta+\sin^2\theta)=K$		
	$\frac{4}{3}ma^2\dot{\theta}\ddot{\theta}+mga(-\sin\theta+2\sin\theta\cos\theta)\dot{\theta}=0$	M1 A1 ft	Differentiating energy equation
	$\ddot{\theta}+\frac{3g}{4a}\sin\theta(2\cos\theta-1)=0$		
	OR $T_1=\frac{\frac{1}{4}mg}{a}(2a+2a\sin\theta)$		
	$T_2=\frac{\frac{1}{4}mg}{a}(2a-2a\sin\theta)$ M1A1		
	$(T_2-T_1)(2a\cos\theta)+mga\sin\theta=I\ddot{\theta}$ M1		
	$(-mg\sin\theta)(2a\cos\theta)+mga\sin\theta=(\frac{4}{3}ma^2)\ddot{\theta}$ A1 ft		
	$\ddot{\theta}+\frac{3g}{4a}\sin\theta(2\cos\theta-1)=0$		
	For small θ, $\sin\theta\approx\theta$, $\cos\theta\approx1$	M1	
	$\ddot{\theta}+\frac{3g}{4a}\theta\approx0$	A1 ft	
	Approx SHM with period $2\pi\sqrt{\frac{4a}{3g}}$ $(=4\pi\sqrt{\frac{a}{3g}})$	M1A1 **8**	
	OR KE is $\frac{1}{2}(\frac{4}{3}ma^2)\dot{\theta}^2$ M1A1		
	$V'(\theta)=mga(-\sin\theta+2\sin\theta\cos\theta)$		
	$V''(\theta)=mga(-\cos\theta+2\cos^2\theta-2\sin^2\theta)$ M2A1		M2 for finding $V''(\theta)$
	$V''(0)=mga$ A1		
	Period is $2\pi\sqrt{\frac{2K(0)}{V''(0)}}=2\pi\sqrt{\frac{\frac{4}{3}ma^2}{mga}}$ M1		*Dependent on M2 above*
	$=2\pi\sqrt{\frac{4a}{3g}}$ A1		